UNIQUENESS PROPERTIES OF GENERAL MONOGENIC FUNCTIONS

BY

WILLIE BUELL EVANS

B.S., Mississippi Southern College, 1939
M.S., Louisiana State University, 1941
M.S., Massachusetts Institute of Technology, 1944

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS IN THE GRADUATE COLLEGE OF THE UNIVERSITY OF ILLINOIS, 1950

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The classical theory of analytic functions of a complex variable is developed with the domain of definition being an open set in the complex plane. If, however, the domain of definition is a closed set, a class of functions arises which is more general than the class of analytic functions. W. J. Trjitzinsky* studied at considerable length functions of a complex variable where the domain of definition is a closed set. In this connection he defined the class of *general monogenic* functions as follows: Let (K) be a simple rectifiable curve forming the boundary of a bounded domain K. Let E be a closed set contained in K + (K). A function $f(z) = u(x,y) + iw(x,y)$ is called general monogenic if u,w, the first partials of u and w and the second partials of u are finite and continuous in E and u is uniform in E; u satisfies Laplace's differential equation; and w is the harmonic conjugate of u in E. The fundamental formulas in the theory of general monogenic functions are, for a in $E \cdot K$,

$$f(a) = h(a) + \iint_K \log(z - a)\, q(x,y)\, dx\, dy \tag{1}$$

and

$$f'(a) = h'(a) + \iint_K \frac{q(x,y)}{z - a}\, dx\, dy \tag{2}$$

where $h(a)$ is analytic in K, and $q(x,y)$ is continuous in $K + (K)$ and equal to zero in E.

Let $f(a)$ be a general monogenic function in E. The non-analytic part of $f'(a)$ was decomposed into the sum of two functions, $W_\nu(a)$ and $R_\nu(a)$. Let AB be a rectifiable Jordan arc in E on which the values of $h'(a)$ and $W_\nu(a)$ are known. Conditions on the rarefication of C(E) were found which will permit a determination of $f'(a)$ in a more extensive set. In this larger set $f'(a)$ is represented as the sum of a single limit of the integral of $h'(a)$ along AB and a double limit of the integral of $W_\nu(a)$ along AB. Also

* W. J. Trjitzinsky, "Theorie des Fonctions d'une Variable Complex Définies sur des Ensembles Généraux," *Annales Scientifiques de L'ecole Normale Superieure*, Vol. 74, 1938, pp. 119-191.

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conditions were found for the rate of vanishing of $q(x,y)$ as the point (x,y) approaches E in order that $f'(a)$ may be represented in a more extensive set.

A similar representation theorem was also proved for functions of the form

$$f(a) = \iint_K \frac{d\mu}{z - a} \tag{3}$$

where μ is an additive set function.

If two general monogenic functions $f(a)$ and $g(a)$ are identical over a denumerable set Λ on an arc AB in E, and a_0 on AB is an accumulation point of Λ from both sides, and C is a point such that the segment a_0C is in E, then conditions were found for the norm of Λ and for the rate at which $q(x,y)$ tends to zero as the point (x,y) approaches E in order to insure the identity of $f(a)$ and $g(a)$ on any closed subinterval contained in a_0C.

VITA

Willie Buell Evans was born on June 5, 1918, in Monticello, Mississippi. After graduating from Monticello High School in 1935, he entered Mississippi Southern College. In June, 1939, he received the Best Citizen Award and the degree of Bachelor of Science, with honors, from that institution. He began his graduate work at Louisiana State University in September, 1939, and held a half-time graduate assistantship for two years and received the degree of Master of Science in 1941. During the academic year 1941-42 he held an instructorship in mathematics at the Agricultural and Mechanical College of Texas. In 1942 he enlisted in the United States Army Air Force and was a cadet in meteorology at Massachusetts Institute of Technology. In September, 1943, upon completing his cadet training he was appointed a Research Assistant in meteorology and received the degree of Master of Science in Meteorology in June, 1944. Upon discharge from the Army in January, 1946, he held an instructorship at Louisiana State University for one semester and then began graduate work at the University of Chicago. In September, 1948, he enrolled in the Graduate College of the University of Illinois. He is a member of Phi Kappa Phi and Sigma Xi.